9th Australasian Natural Convection Workshop (9ANCW)
Monash University, Melbourne, Australia, 14-15 December 2015

Stability of Rotating Cylinder Driven by Radial Horizontal Convection

Wisam K. Hussam*, TzeKih Tsai and Gregory J. Sheard

Department of Mechanical and Aerospace Engineering, Monash University, VIC 3800, Australia

*wisam.alsaadi@monash.edu

1. INTRODUCTION

A natural convection flow develops as a result of the horizontal temperature differences, and if the temperature difference is sufficiently large, unsteady over-turning circulation occurs [1]. Many studies have focused on planar horizontal convection where the effects of thermal heating and aspect ratio are being investigated in relation to heat transfer characteristics. However, in this study we focus on the effect of rotation on horizontal convection flows, which are important in many industrial applications as well as in geophysical flows, including circulations of ocean and atmospheres. We employ linear stability to predict the instability modes of the flow, and interrogate the dominant instability modes.

2. NUMERICAL MODEL AND METHODOLOGY

The system under consideration consists of a free surface cylindrical enclosure rotating with an angular velocity Ω. It is filled with a fluid with Prandtl number Pr = 6.14, consistent with water at laboratory conditions, and a temperature profile varying radially is imposed over the base. The tank radius R and height H combine to define an aspect ratio $AR = H/R$, which in this study is fixed at AR = 0.4. The governing equations for this system are the Navier-Stokes and energy equations for linear Boussinesq fluid [3]. These equations are computed on a two–dimensional axisymmetric domain using a high-order in-house solver, which employs a spectral element method for spatial discretzation and a third-order time integration scheme based on backwards differencing. The Rayleigh number characterizes the strength of thermal forcing and is defined as $Ra = g\alpha\delta TR^3/\nu k$, where g is gravitational acceleration, ν is the kinematic viscosity, and k_T is the thermal diffusivity of the fluid. In a rotating system the ratio between thermal boundary layer thickness and Ekman layer thickness is important in describing the flow. The square of the ratio between these two thickness scales is $Q = 1/Ek\ Ra^{2/5}(H/R)^{-2}$, where Ek is an Ekman number characterizing the ratio of viscous to Coriolis forces.

The linearised governing equations are obtained by decomposing the velocity, pressure and temperature into the sum of axisymmetric filed and small non-axisymmetric perturbation. The perturbation field is constructed as a single complex Fourier mode of an azimuthal expansion of the flow field and the wavenumber of the perturbation is a parameter in the stability analysis.

3. RESULTS

Figure 4 plots the marginal stability curve for $Q \leq 60$. Everywhere inside the marginal stability curve, the growth rate is positive and hence the flow is unstable to infinitesimal disturbances of the enclosed azimuthal wavenumbers. Five distinct instability mode branches have been identified in the Q–k parameter space. Below Q = 1.86 the flow is stable. At Q = 1.86, instability first emerges with an azimuthal wavelength $k \approx 5$. The domain of unstable wavenumbers widens rapidly with increasing Q up to $Q \approx 12$, with instability growth predicted across $0 \leq k \leq 80$. The band of unstable wavenumbers contracts to $k \approx 48$ at $Q \approx 20$, and a gradual monotonic increase is observed thereafter, passing $k \approx 64$ at Q = 60. The loci of maximum growth rate in figure 4 reveal that at small Q only a single instability mode is predicted, but a second branch emerges at smaller wavenumbers (i.e. $k \approx 2$) at $Q = 4.5$. Thereafter, a single branch persists with dominant wavenumber increasing from $k \approx 20$ to $k \approx 30$ as Q increases from 20 to 30. The asymptotic state for high Q appears to be a single instability mode branch of more modest wavenumber, ranging from $k \approx 15$ at $Q = 30$ to $k \approx 30$ at $Q = 60$.

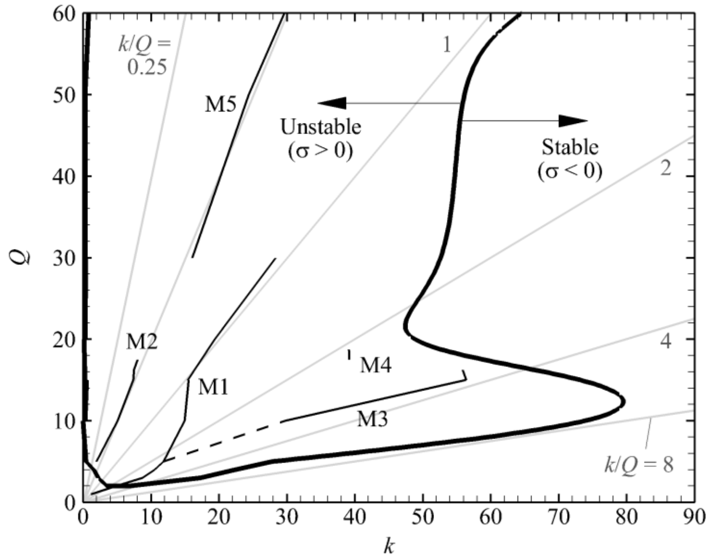

Figure 1. Neutral stability (solid line) and loci of maximum growth rate (dashed lines) for the five identified mode branches (labelled) across the Q–k parameter space for $Ra = 10^9$ and $Pr = 6.14$. For guidance, the data is plotted over faint radial lines of constant k/Q, with values doubling in the clockwise direction from k/Q = 0.25 to 8.

4. CONCLUSIONS

Multiple instability modes have been identified in a rotating cylinder driven by a horizontal convection. It is found that a single instability mode with modest wavenumbers is dominated for high rotation rate.

REFERENCES

[1] Hughes, G.O. and Griffiths, R.W., Horizontal Convection, *Annu. Rev. Fluid Mech.*, **40**, 2008, 185–208.

[2] Sheard, G. J. & King, M. P. 2011 Horizontal convection: Effect of aspect ratio on Rayleigh number scaling and stability. *Appl. Math. Model*. **35** (4), 1647–1655.

[3] Mullarney, Julia C, Griffiths, Ross W & Hughes, Graham O 2004 Convection driven by differential heating at a horizontal boundary. Journal of Fluid Mechanics 516, 181–209.

[3] Hussam, W. K., Tsai, T. K. & Sheard, G. J. 2014 The effect of rotation on radial horizontal convection and Nusselt number scaling in a cylindrical container. *Int. J. Heat Mass Trans*. **77**, 46–59.

[4] Hignett, P., Ibbetson, A. & Killworth, P. D. 1981 On rotating thermal convection driven by non-uniform heating from below. *J. Fluid Mech*. **109** (1), 161–187.