Criticality and structure of the asymmetric vortex shedding modes of bluff ring wakes

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A numerical study of the asymmetric vortex shedding wakes of bluff rings is presented. A useful feature of the bluff ring system is that by varying a single geometric parameter – the aspect ratio (*Ar*) – a diverse set of geometries are defined. The aspect ratio is a ratio of the mean ring diameter (*D*) to the ring cross-section diameter (*d*), such that $Ar = D/d$. An aspect ratio of $Ar = 0$ defines a sphere, $Ar > 1$ defines open rings (i.e., rings with a hole at the axis), and for aspect ratios $Ar \rightarrow \infty$, the wake of the ring cross-section locally approaches the wake of the straight circular cylinder. Rings with $Ar > 4$ comprise the scope of the present study, as the wakes of these rings undergo an axisymmetric Hopf transition prior to an asymmetric transition.

A spectral-element method is employed to compute the wakes behind the bluff rings. High-order Lagrangian tensor-product polynomials are used within each element, with node points corresponding to the collocation points for efficient Gauss-Lobatto-Legendre integration. A finite-element method links the macro-elements comprising the mesh. The azimuthal periodicity of the bluff ring geometry allows a Fourier expansion of the spectral-element mesh in the azimuthal direction, simplifying the computation of asymmetry in the wake.

A linear stability analysis of the axisymmetric vortex shedding wakes of bluff rings (Sheard *et al.* 2001) has previously predicted the existence of Mode A and Mode B type asymmetric vortex shedding modes, consistent with the vortex shedding wake of the straight circular cylinder. Interestingly, a third instability mode is also predicted, with spatio-temporal symmetry characteristics similar to the subharmonic mode observed in the perturbed circular cylinder wake (Zhang *et al*. 1995), and predicted in the wake of the square cylinder (Robichaux, Balachandar & Vanka 1999).

The present study computes the asymmetric wake structures of the three predicted vortex shedding modes, and iso-surface plots are included to identify the vorticity distribution of the modes. A further analysis of the criticality of the asymmetric modes is achieved through an application of the truncated Landau equation. Henderson (1997) successfully applied the Landau equation to the Mode A and Mode B transitions of the straight circular cylinder wake, and determined that their transitions were subcritical and supercritical, respectively. We verify that the same criticality is found for the corresponding modes of the bluff ring wakes, and determine that the Mode C transition occurs through a supercritical bifurcation. This is the first instance that the criticality of such a subharmonic vortex shedding mode has been reported to the knowledge of the authors.

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