Hydraulic Analogy Examination of Underexpanded Jet Shock Cells using Reference Image Topography

V. Kumar¹, I. Ng¹, G. J. Sheard², K. Hourigan¹,² and A. Fouras¹,²

¹Division of Biological Engineering, Monash University, VIC 3800, Australia
fouras@eng.monash.edu.au

²Fluids Laboratory for Aeronautical and Industrial Research (FLAIR), Department of Mechanical and Aerospace Engineering, Monash University, VIC 3800, Australia

ABSTRACT

This paper examines the shock cell structure at the exit of an underexpanded jet nozzle using a hydraulic analogy. This approach allows thorough investigation of this important flow regime at a greatly reduced expense. Applying the novel Reference Image Topography technique (Fouras et al. [1]), the wavy water surface is reconstructed, providing a pressure map of the nozzle exit region. This allows a detailed analysis of the underlying shock cell structure and the flow instabilities resulting in jet screech.

1. INTRODUCTION

The flow behind jet nozzles has been a matter of great interest due to its effects on the aerodynamics, structure and propulsion system of an aircraft. The first shock cell formed at the exit of an underexpanded jet nozzle provides valuable information regarding the flow properties, such as pressure variations, exhaust velocity, acoustic vibrations, viscous effects, etc. A thorough understanding of pressure distribution at the exit of the nozzle can assist in various studies, such as the investigation of screech phenomenon that is potentially harmful for aircraft bodies (Brocher et al. [2]; Buchanan et al. [3]), and enhanced mixing for combustion applications (Alkislar et al. [4]; Alkislar et al. [5]). A nozzle is said to be underexpanded when the ambient atmospheric pressure is lower than the pressure at the nozzle exit, which implies that the flow at the exhaust is capable of further expansion.

The technique used to visualise this flow is based on the concept of comparing a reference image and the same image through a distorted surface to produce apparent displacement vectors that can be used to reconstruct the surface topography. Inherently, this is a simple, cost-effective and highly accurate technique.

2. THE HYDRAULIC ANALOGY

According to the hydraulic analogy, the free-surface topography of water on a water table relates to the pressure, temperature and density profiles, while the velocity field is directly related to vorticity in the flow (Buchanan et al. [3]; Fouras et al. [6]). The hydraulic analogy is useful for simulating high-speed 2D compressible ideal gas flow with shallow water flow (Buchanan et al. [3]). The study of high-speed jet flows becomes much simpler, more convenient and accurate, as the wave propagation and flow speed is three orders of magnitude slower than gas flow. Differences in pressure, temperature and density in gas flow correspond to differences in water depth across a converging nozzle placed on the water table. The Froude number, $Fr$, in the water is considered analogous to the Mach number, $Ma$, for gas flow (Buchanan et al. [3]), subsequently allowing the pressure ratio across the nozzle to determine the equivalent speed of the gas (White [7]). Dynamic equivalence between the two flows is thus achieved by a simple manipulation of equations (as in (1) and (2)).

$$Ma = Fr = \left[2 \left(\frac{h}{h_0} - 1\right)\right]^{\frac{1}{2}} \gamma = 2,$$  

(1)

$$\frac{p}{p_0} = \left[1 + \frac{1}{2}(\gamma - 1)Ma^2\right]^{\frac{1}{\gamma}} \gamma = 2,$$  

(2)

where $\gamma$ is the specific heat ratio, $h$ is the water depth in the upstream reservoir and $h_0$ is the mean water depth in the test section. A summary of the hydraulic analogy is provided in Table 1 (Buchanan et al. [3]).

Table 1: Analogous equations and variables for two dimensional gas and shallow hydraulic flows.

<table>
<thead>
<tr>
<th>Gas flow</th>
<th>Hydraulic flow</th>
<th>Implication</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\frac{T}{T_0} = 1 + \frac{1}{2}Ma^2$</td>
<td>$\frac{T}{T_0} = 1 + \frac{1}{2}Fr^2$</td>
<td>$\frac{L}{L_0} \equiv \frac{R}{R_0}; \gamma = 2$</td>
</tr>
<tr>
<td>$\rho \left(\frac{pu}{h} + \frac{hv}{y}\right) = 0$</td>
<td>$\rho \left(\frac{hu}{h} + \frac{hv}{y}\right) = 0$</td>
<td>$Ma \equiv Fr$; $\frac{p}{p_0} \equiv \frac{h}{h_0}$</td>
</tr>
<tr>
<td>$\frac{p}{p_0} = \frac{R}{R_0} \left(\frac{h}{h_0}\right)^2$</td>
<td>$\frac{p}{p_0} = \frac{R}{R_0} \left(\frac{h}{h_0}\right)^2$</td>
<td>$\gamma = 2$</td>
</tr>
</tbody>
</table>

It is worth noting that the analogy does not provide a direct quantitative representation of aerodynamic flows in air, but rather that of a hydraulic gas in which $\gamma$ is 2. However, this restriction can be neglected as most relevant experiments will have insignificant influence of varying $\gamma$ (Buchanan et al. [3]).

3. FLOW VISUALISATION

3.1 Technique

The technique used in this paper is based on the Reference Image Topography technique described in Fouras et al. [1]. A converging perspex nozzle is placed on a glass-bottomed water table, through which shallow water flows at a steady rate. A ground glass plate, used as a reference object, is attached underneath the glass bottom in the region of the shock cell. The camera’s frame of view (105 mm streamwise × 85 mm lateral = 1024 pixels × 826 pixels) encompasses the entire first shock cell. A light source (300 W halogen lamp), collimated by a long aspect ratio box lined with a reflective surface, is used to illuminate the semi-translucent reference object. A similar technique was employed by Moisy et al. [8].
3.2 Height Estimation Algorithm

The apparent displacement vectors ($\delta_\delta, \delta_\xi$) relate to the free-surface gradients ($\partial H/\partial x$ and $\partial H/\partial y$), which can be integrated by an algorithm to estimate the water height distribution (Fouras et al. [6]). The mean water height, set as the reference height ($H_{ref}$), is taken as the initial height estimate, $H$. Based on the configuration in Figure 2 and using Snell’s law of refraction at the water-air interface ($\eta_{air} \sin \alpha_1 = \eta_{liquid} \sin \alpha_2$), the relationship to estimate $\partial H/\partial x$ and $\partial H/\partial y$ using the current guesses of $H$ is derived as (Fouras et al. [6]):

$$\tan^{-1} \frac{\delta_\delta}{H} = \tan^{-1} \left( \frac{\partial H}{\partial x} \right) - \sin^{-1} \left[ \frac{\eta_{air}}{\eta_{liquid}} \sin \left( \tan^{-1} \frac{\partial H}{\partial x} \right) \right].$$

Integrating $\partial H/\partial x$ and $\partial H/\partial y$ yields the estimated heights, $H_{adj}$, which can be adjusted to give $H_{adj'}$ by applying a minimum-error approach between measured displacements and reference image particle positions. An iterative process is applied until the value of $H_{adj'}$ converges. A detailed description of the height-solving algorithm is contained in Fouras et al. [6]. As the distance of the camera from the glass table is much greater than the thickness of the glass bottom and the angle of refractions at the glass-air and glass-water interfaces are very small, the refraction effect of the glass bottom can be neglected (Moisy et al. [8]).

3.3 Shock Cells

In the 2D visualisation of Figure 3, the nature of the jet shock cells in a supersonic flow regime can be seen. A series of expansion waves are formed at the nozzle exit that propagate until they reach the jet shear (boundary) layer. Here they are reflected as compression waves and so on until the ambient pressure (or back pressure) is reached. This process of reflection of expansion and compression waves results in a diamond-shaped pattern being formed in the jet. Larger Mach numbers produce greater shock cell spacing.

4. RESULTS

The upstream water depth was varied to control the pressure ratio and hence the resulting Mach number (Table 1). A low downstream water depth was chosen to produce well-defined shock cells with relatively low surface gradients for effective height estimation. An interrogation window size of 16 pixels × 16 pixels was chosen to provide high accuracy and effective vector correlation in an efficient manner.

4.1 Time-averaged

The time-averaged surface reconstruction for early-development shock cells is shown in Figure 4. The harmonics are clearly evident in the reconstruction, with greater pressure gradients visible at the shock cell corners. The time-averaged surface displays flat edges in the jet shear layer along the side of the shock cells, while the shock cells remain stable and steady in shape. A gradual decay in the strength of the shock cells can be observed with distance downstream.
Figure 4: Time-averaged surface reconstruction of the shock cells. The variation in water surface height in the region of the first four shock cells is distinct.

Figure 5: Downstream propagation of vortices along jet shear layer on either side of shock cells. The vortex structures can be seen to increase in size and strength as they propagate downstream. $\Delta t = \frac{1}{14}$ s. A relatively stable surface is observed near the nozzle exit, while interaction between shock cells and vortices downstream creates instabilities. Flow is from left to right.
4.2 Temporal

The time-averaged surface (Figure 4) is subtracted from the instantaneous surfaces to highlight the non-steady regions of the flow. This is shown in Figure 5, where periodic vortices propagate downstream on either side of the shock cells. Over time, these vortices are averaged to the mean water height, as exhibited by the flat regions in Figure 4. It is hypothesised that these vortex structures pump fluid into and out of the jet shear layer (Woodmansee et al. [9]).

These vortex structures originate from Taylor-Görtler instabilities caused by the concave curvature in the streamwise direction around the first shock cell in the expanding jet (Woodmansee et al. [9]; Schlichting et al. [10]; Smits et al. [11]). This is well in agreement with the results, where the vortices can be seen to originate from the first shock cell and the vorticity increases in strength downstream. Interaction between the larger vortices and shock cells downstream produces instabilities in the central flow region, which can be linked to the generation of acoustic power at the screech frequency (Buchanan et al. [3]). The strength of these vortices is purported to be related to the screech-tone intensity (Alkislar et al. [4]). The aeroacoustic phenomenon of screech, being potentially harmful for aircraft bodies, is an area of continual research and investigation (Buchanan et al. [3]; Alkislar et al. [4]).

5. CONCLUSIONS

The shock cell structure at the exit of an underexpanded jet nozzle has been analysed using the hydraulic analogy. Using the Reference Image Topography technique, the topography of the water was reconstructed. Vortices were observed to propagate downstream on either side of the shock cells with interaction downstream as the vortices grew in size. The variation in water height is intimately related to the pressure, temperature and density fields of the flow, which play a significant role in relevant fluid mechanics, jet propulsion and aircraft structural studies. The technique employed is relatively simple, cost-efficient, accurate, of superior image quality and easily repeatable (Fouras et al. [1]). Further work in analysing the velocity and vorticity distributions in the region, together with the pressure and temperature distributions, will provide a better understanding for the development of appropriate control measures for jet screech.

REFERENCES


