9th Australasian Natural Convection Workshop (9ANCW)
Monash University, Melbourne, Australia, 14-15 December 2015

Linear stability of horizontal, fully developed, quasi-two-dimensional liquid metal duct flow under a transverse magnetic field and heated from below

Tony Vo[1*], Alban Pothérat[2] and Greg Sheard[1]

[1]The Sheard Lab, Department of Mechanical and Aerospace Engineering, Monash University, VIC 3800, Australia
[2]Applied Mathematics Research Centre, Coventry University, Priory Street, Coventry CV15FB, United Kingdom
*tony.vo@monash.edu

1. INTRODUCTION

This abstract describes the linear stability of an electrically conducting fluid flowing through a rectangular duct under a transverse magnetic field subjected to heating from below and cooling from above. Examples of these flows are found in the casting of steel slabs [1] and in self-cooled liquid metal blankets used in magnetic confinement fusion reactors [2]. The primary motivator in the later example is to enhance the heat transfer from the inner wall of the blanket which is heated from the magnetically confined plasma. This can be achieved by adding turbulence promoters such as bluff bodies [3] or an external current injection [4]. An alternative to enhancing the heat transfer is to cause the flow to become unstable. This paper investigates the instability structures of the flow.

2. NUMERICAL SETUP

An electrically conducting fluid flowing in a rectangular duct is subjected to a transverse magnetic field. The magnetic field suppresses motions that are parallel to the magnetic field lines causing the flow to become quasi-two-dimensional which can be modelled by the SM82 model [5]. A schematic of the problem is provided in Figure 1. Furthermore, the velocity and temperature solution of the base flow is assumed to be independent of the streamwise direction (x-direction) and therefore, the base flow solution simplifies to be one dimensional (only a function of y).

The velocity profile of the base flow is dependent on the strength of the magnetic field which changes from a parabolic Poiseuille solution (no magnetic field) to a Hartmann solution (strong magnetic field). The Hartmann solution is described by an exponential profile in the sidewall boundary layers (Shercliff layers), and a uniform interior core as shown in Figure 1. A linear temperature profile is taken to be the scalar base flow solution across the duct height (y-direction) with hot temperature at the bottom wall and cooler temperature at the upper wall.

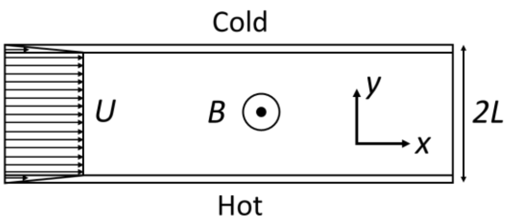

Figure 1. A schematic of the one-dimensional base flow being investigated.

The governing parameters of the problem are characterised by a Reynolds number Re, a Rayleigh number Ra, and a modified Hartmann number H. Here, $Re = 0$ (no through flow), $Ra = 0$ (no heating) and $H = 0$ (no magnetic field) represent special flow conditions. A Prandtl number of $Pr = \nu/\kappa = 0.022$ is used which is representative of Galinstan (GaInSn) liquid metal. The linearised governing equations are treated as an eigenvalue problem where the velocity and temperature perturbations are solved using an eigenvalue solver.

3. RESULTS

The neutral stability curves of *Ra* against *H* are obtained by searching for eigenvalues corresponding to zero growth rates. The results are shown in Figure 2. The flow conditions located above and to the left of the curves represent unstable regions. At *Re* = 0, the neutral curve demonstrates a single continuous instability branch spanning over the entire range of *H* investigated. This curve changes its profile and eventually breaks into two instability branches when *Re* is increased to 350. Clearer illustrations are seen for *Re* > 350. Overall, the plot illustrates an increased stability with increasing *Re* up to *Re* = 1e3. This suggests that the through flow acts to inhibit the thermal instability.

At *Re* = 5e3, the neutral stability curve which spans over the lower values of *H* only begins to deviate away from the horizontal (constant *Ra*) with increasing *H*. This trend is clearer at *Re* = 1e4 and *Re* = 1e5 where the noted instability branches have become vertical. This change is due to the shear instability becoming more dominant compared to the thermal instability. Therefore, flows are found to be always unstable regardless of *Ra* provided that there is a sufficiently large *Re* coupled with a sufficiently small *H*.

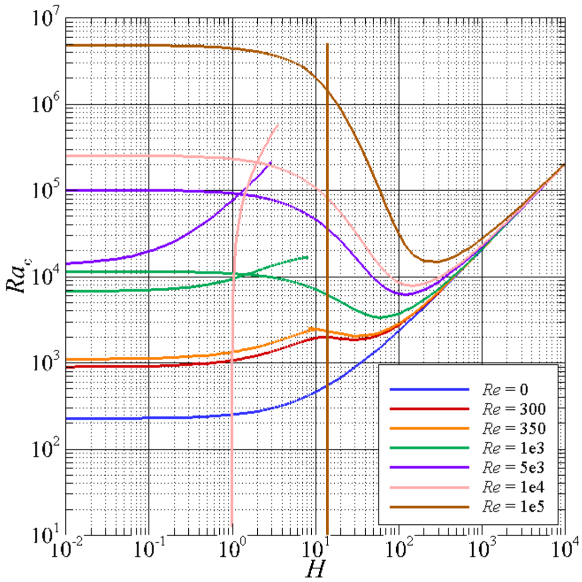

Figure 2. Neutral stability curves of various *Re* as a function of *Ra* and *H*.

4. CONCLUSIONS

Multiple instability modes have been identified in an electrically conducting fluid which flows through a rectangular duct subjected to a transverse magnetic field and heating. From the neutral stability curves, the flow is found to transition from a single instability mode at *Re* = 0 to two modes at higher *Re*.

REFERENCES

[1] Cukierski, K. & Thomas, B. G. (2008) Flow control with local electromagnetic braking in continuous casting of steel slabs. *Metall. Mater. Trans. B*, **39** (1), 94-107.

[2] Müller, B. & Bühler, L. (2001) Magnetofluiddynamics in channels and containers. Springer-Verlag Berlin Heidelberg.

[3] Hussam, W. K., Thompson, M. C. & Sheard G. J. (2011) Dynamics and heat transfer in a quasi-two-dimensional MHD flow past a circular cylinder in a duct at high Hartmann number. *Int. J. Heat Mass Transfer*, **54**, 1091-1100.

[4] Bühler, L. (1996) Instabilities in quasi-two-dimensional magnetohydrodynamic flows. *J. Fluid Mech.*, **326**, 125-150.

[5] Sommeria, J. & Moreau, R. (1982) Why, how, and when, MHD turbulence becomes two-dimensional. *J. Fluid Mech.*, **118**, 507-518..