The development of unsteady axial and transverse forces on a cylinder with free hemispherical ends

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Abstract

The fluid forces imparted on a cylinder with free hemispherical ends are investigated by comparing the flow past a cylinder with a short length ratio to a sphere.

The development of time-dependent flow initiates both axial and transverse modes of oscillation in the wake of the cylinder, whereas for a sphere oscillation is only observed in one plane. The relationship between these modes to the first-occurring Hopf mode in the wake of a sphere is considered.

The Hopf transitions in the cylinder wakes are analyzed with the Landau equation. It is found that the transitions occur through supercritical bifurcations, and the bifurcation process is initiated by a transverse mode even at short length ratios.

Key words: Computational fluid dynamics, Spectral-element method, Cylinder, Hopf transition, Strouhal number, Reynolds number

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1 Introduction

The flow normal to a cylinder with hemispherical ends has met with attention in recent years [1][2]. Interest in the study of this body is predominantly due to the ability of the body to represent a sphere at the limit of small length ratio, while approaching a straight circular cylinder at large length ratios. The length ratio L_R is defined as a ratio of cylinder length to diameter (see Figure 1).

The Strouhal–Reynolds number profiles of cylinders with length ratios in the range $1 \leq L_R \leq 5$ have been recorded by Schouveiler & Provansal [1], and Provansal, Schouveiler & Leweke [2]. They showed that over this range of short length ratios, the critical Reynolds number for the onset of unsteady flow increase with a decrease in length ratio, suggesting a link in the first-occurring Hopf bifurcation in the wake between a straight circular cylinder ($Re_c \approx 47$ [3][4]) and a sphere ($Re_c \approx 272$ [5][6]). Consistent with studies of the flow past spheres and circular cylinders, a Reynolds number $Re = Ud/\nu$ is defined for the flow past cylinders with hemispherical ends based on the diameter d, as well as the freestream velocity U and kinematic viscosity ν.

A complication for the bifurcation process leading to unsteady flow past cylinders with hemispherical ends is that the Hopf bifurcation in the wake of a sphere is preceded by a regular bifurcation to non-axisymmetric flow [7][6]. The existence of a similar regime of steady asymmetric flow has not yet been reported for cylinders with short length ratios.

In the present study the flow past a cylinder with $L_R = 2$ is compared to the computed wake behind a sphere. In a manner similar to studies by Provansal, Mathis & Boyer [3], Thompson, Leweke & Provansal [6] and Sheard, Thompson & Hourigan [8], coefficients of the Landau model are determined to analyze the non-linear evolution of the instability modes in the wake. For details the aforementioned references should be consulted. Briefly stated, a plot of $\mathrm{d}\log|A|/\mathrm{d}t$ against $|A|^2$ (where $|A|$ is the mode amplitude, here taken from pressure force measurements at the cylinder surface) can be used to determine the linear growth rate of the instability by extrapolating the data to $|A|^2 = 0$, and the slope in the vicinity of $|A|^2 = 0$ determines the potential of the mode to exhibit hysteresis. A hysteretic (i.e., discontinuous or subcritical) bifurcation is predicted by a positive gradient, and a non-hysteretic (i.e., continuous or supercritical) bifurcation is predicted by a negative gradient.

2 Numerical formulation

The flow is computed using a cylindrical-polar formulation of a spectral-element method that has been successful in computing the low-Reynolds-number flows past both spheres [6] and rings [9][8]. Details of the mesh formulation and grid independence studies may be found in [10][11], but a summary is included here.

The numerical formulation is somewhat unique in that a Fourier expansion is employed to discretize the flow about the symmetry axis of the body, but instead of computing the flow in a direction parallel to the symmetry axis as is standard for sphere computations [7][6], the flow is computed normal to the symmetry axis (and the cylinder).

Careful attention was paid to mesh development to ensure that adequate resolution was obtained using the “crossflow” mesh formulation. A sphere was computed with the present family of meshes, and the pressure and viscous drag components where predicted to within 1% accuracy when compared with accurate computations from previous studies (e.g., TombOrsz00).

3 Comparing sphere and short cylinder wakes

The wakes behind both a sphere ($L_R = 1$) and a cylinder ($L_R = 2$) were computed at $Re = 300$, which is well above the critical Reynolds number for transition to unsteady flow in the wake of a sphere, as well as being greater than the approximate critical Reynolds number observed for a cylinder with $L_R = 2$ in experiment ($Re_c \approx 155$).

Not surprisingly, the wake behind a sphere developed the familiar unsteady hairpin vortex pattern [12][13]. It is pertinent to note that as testament to the well-resolved flow in the vicinity of the body surface, the orientation of the periodic wake was not aligned with the major axes of the computational grid.

The development of unsteady flow in the wake of the cylinder was both complicated and interesting. As the cylinder geometry places limitations on the orientation of any asymmetric modes in the wake, components of the fluid force acting in both the axial (parallel to the cylinder axis) and transverse (normal to the cylinder and flow) directions were monitored. The plot in Figure 2 shows the evolution of asymmetric modes in both directions, following the prior development of a periodic zero-mean unsteady oscillation in the transverse direction. The next section details a Landau analysis of the first-occurring transverse Hopf bifurcation.

4 The first-occurring Hopf bifurcation

In the wake past a straight circular cylinder, a Hopf bifurcation develops in the transverse direction. When computed from a steady-state initial condition, it is observed that unsteady flow develops in the transverse direction at $Re = 225$ for a cylinder with $L_R = 2$.

The variation in the fluid force on the cylinder was recorded as a measurement of the amplitude of the Hopf bifurcation in the wake. The transverse oscillation that evolved was initially symmetrical about the cylinder, but almost immediately upon saturation of the mode this symmetry was broken, and the mode developed a non-zero mean. To determine the non-linear bifurcation characteristics, the time history of the transverse mode evolution was used to generate the plot shown in Figure 3. The plot verifies that the initial transverse bifurcation occurs through a continuous supercritical bifurcation, with a linear growth rate of $\sigma_T \approx 0.036$.

5 Subsequent modes in the wake

As the plot in Figure 2 shows, the development of transient asymmetry in the mean flow develops rapidly, and in addition an axial asymmetry also evolves. This asymmetry evolves as a regular bifurcation, but becomes unsteady upon saturation. A similar analysis to that in the preceding section verified that this regular mode also occurs through a supercritical bifurcation. It is interesting to note that once the modes reached an equilibrium, the axial mode had a non-zero mean, while the transverse mode returned to an approximately zero mean.

In Figure 4 plots of the wake are shown at each of the major transition points in the bifurcation process.

6 Conclusions

The bifurcation process leading to unsteady flows in both the axial and transverse directions has been analyzed for a cylinder with free hemispherical ends and a length ratio of $L_R = 2$ at $Re = 225$. The observation that the first-occurring symmetry-breaking bifurcation is a transverse Hopf bifurcation as in the wake of a straight circular cylinder leads to the conclusion that smaller length ratios should be studied to determine if a steady asymmetric wake is found prior to the development of unsteady flow for any cylinders with $L_R > 1$.

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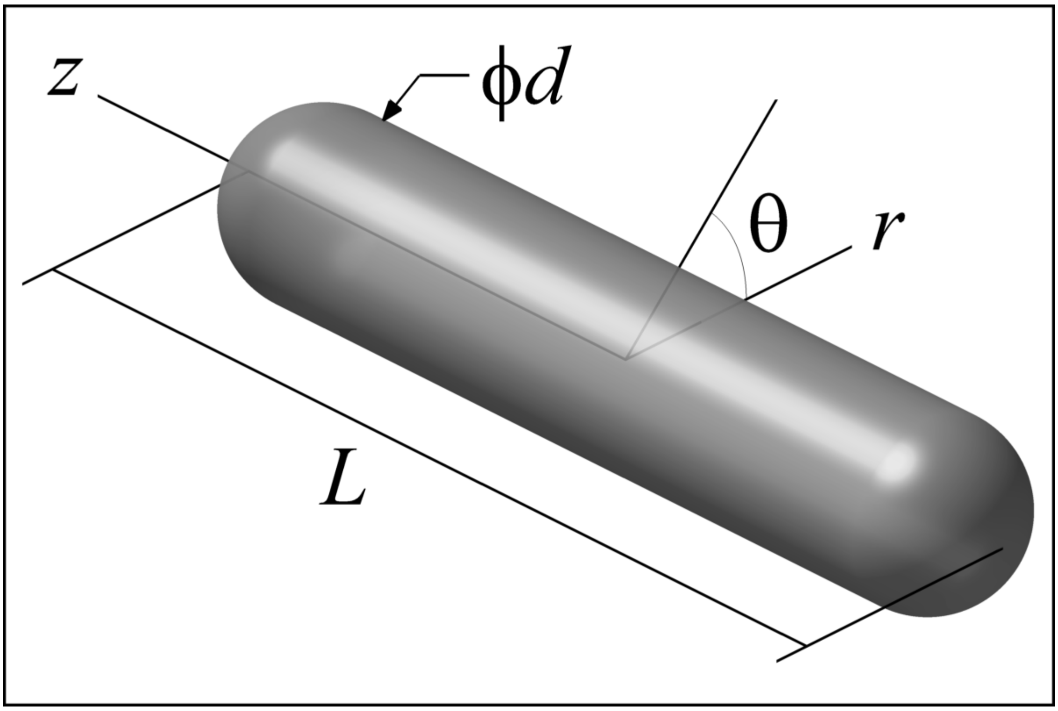

Fig. 1. The coordinate system relative to a cylinder with free hemispherical ends. The direction of flow is across a plane of constant z, and the length ratio $L_R = L/d$.

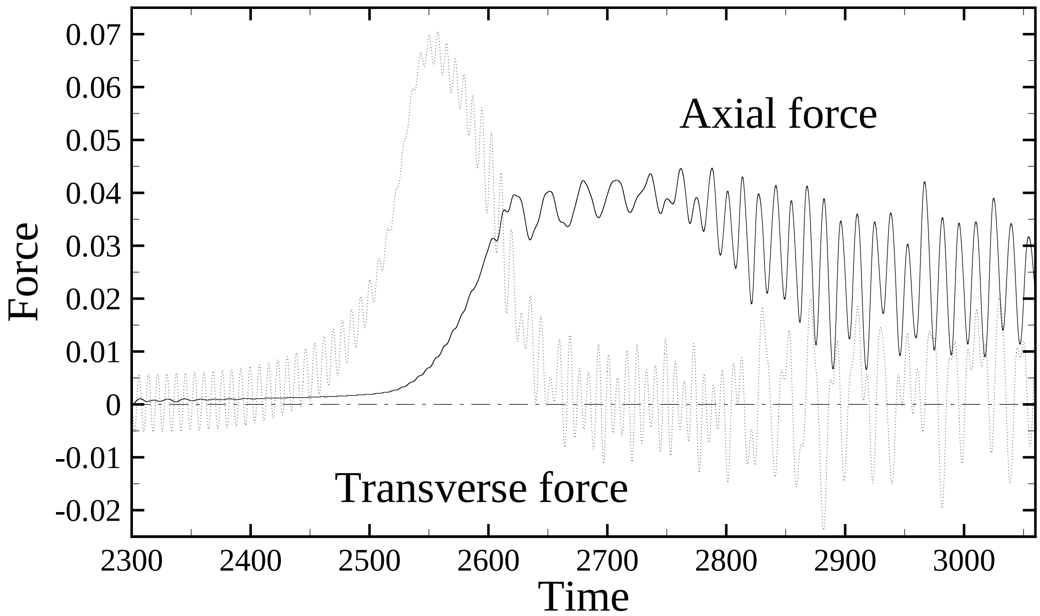

Fig. 2. A plot of the time-history of the evolution of unsteady axial and transverse modes in the wake of a cylinder with $L_R = 2$ at $Re = 225$.

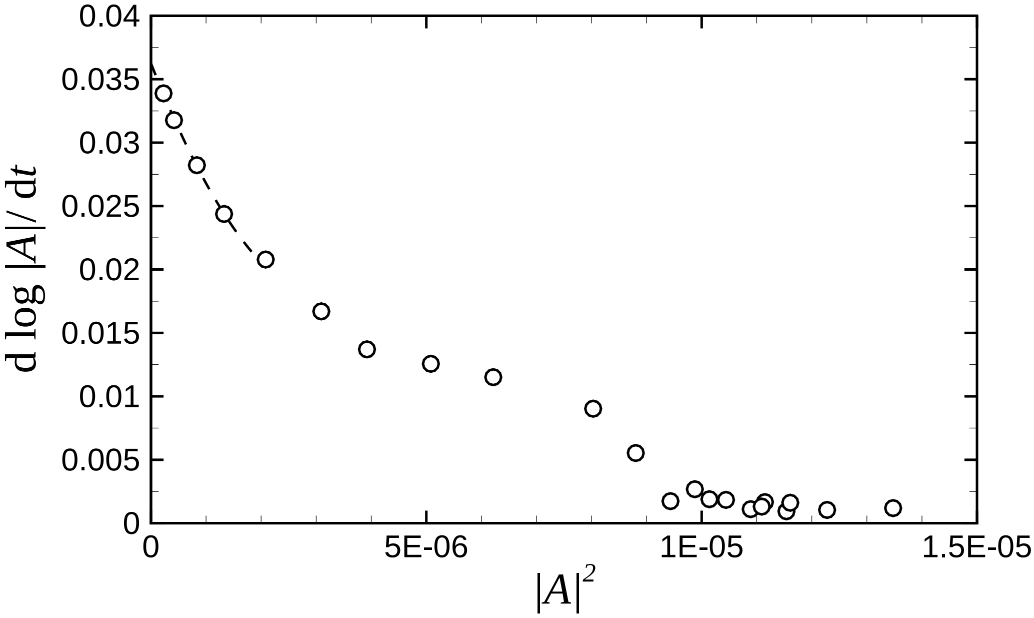

Fig. 3. The variation in growth rate with $|A|^2$ for the transverse Hopf mode in the wake of a cylinder with $L_R = 2$ at $Re = 225$. The negative slope in the vicinity of the ordinate axis verifies that the transition is supercritical. A polynomial extrapolation to $|A|^2 = 0$ yields a linear growth rate of $\sigma_T \approx 0.036$.

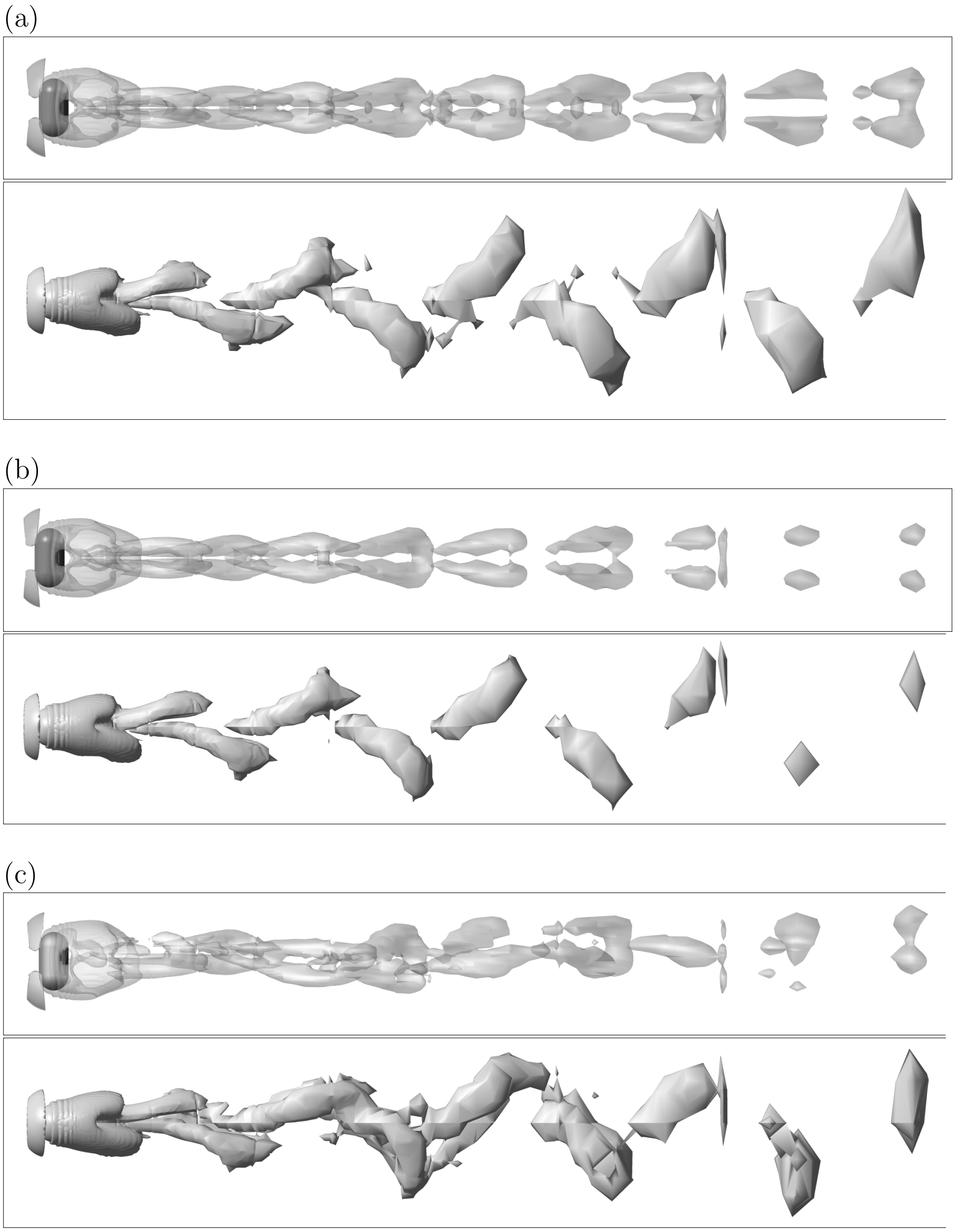

Fig. 4. Isosurface plots showing the alteration in wake structure as the transverse and axial Hopf bifurcations evolve in the flow. The wake is shown (a) prior to the development of a non-zero mean transverse force oscillation, (b) at the point of maximum mean transverse force, and (c) with both axial and transverse Hopf modes saturated.