

## Sources of Perturbation Growth in Cylinder Wake Instabilities

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### Abstract

Flows past cylindrical obstructions have been the subject of keen research interest owing to their ubiquity, importance to engineering structures under aero- or hydrodynamic load, and the fascinating flow patterns they produce. These flows undergo a systematic sequence of transitions, each of which incur a change in their dynamics. This study focuses on the instabilities associated with bifurcations to three-dimensional flow. Although these transitions have been investigated extensively in the past (see report by Barkley & Henderson<sup>1</sup> and review by Williamson<sup>2</sup>), this study approaches the problem from an energetics perspective through the consideration of the out-of-plane averaged perturbation kinetic energy (PKE) equation in an attempt to understand the sources feeding the growth of the linear modes.

For this study, two-dimensional base flow solutions are computed from the incompressible Navier—Stokes equations evaluated at the transition Reynolds numbers for modes A and B. Spatial discretisation is achieved through a nodal spectral-element method, and the equations are evolved forward in time using a third-order-accurate time integration scheme based on backwards differentiation<sup>3</sup>. The linearised equations are also evolved using the same numerical scheme, and the Floquet modes determined using an Arnoldi package (ARPACK). Terms in the out-of-plane averaged PKE equation are then computed.

The oral presentation will discuss the contributions of each term of the PKE equation toward the instability growth, comparing the cases for both modes A and B. The period- averaged terms of the total PKE equation show that the perturbation growth in both modes A and B are predominantly accelerated through terms containing the cross-flow gradient of the base-flow velocity, while viscous dissipation dampens its growth as expected. A key difference between the mean energy exchange rates of the two modes are in the magnitudes of each term’s contribution relative to the total perturbation kinetic energy; mode A draws PKE at a significantly lower rate than mode B. Dynamically, each contributing term for mode A either distinctly accelerates or delays PKE growth over the shedding cycle, unlike mode B where PKE production through streamwise gradients of the base flow switches between phases of promoting and impeding the instability growth, albeit at negligible amplitudes.

The locus of the dominant production terms' maxima for mode A locate its origins in the core of the forming vortex in the wake, and it develops according to the description provided in Thompson *et al.*<sup>4</sup> in demonstrating the co-operative elliptical instability mechanism for mode A. For mode B, the maximum production rate occurs within the highly strained region of the near wake. As the vortex is being shed, these regions of high production rate are drawn into the forming vortices, facilitating the feedback reflected in the symmetry of mode B. Contributions from these terms appear negligible downstream where the vortices are relieved from most of the strain induced from the shedding. The perturbations then weaken primarily due to the action of viscosity unlike mode A where the transport terms carry some perturbation out of the domain.

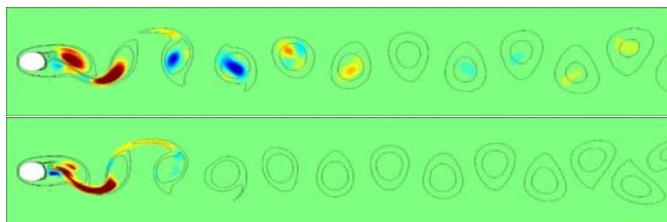


Figure 1: Distribution of the dominant production terms ( $u'v'dU/dy + v'v'dV/dy$ ) corresponding to events of the base flow. Red (blue) contours indicate net positive (negative) contribution to the PKE growth rate, and the solid lines draw the vortices in the two-dimensional vortex street of the base flow. The top image corresponds to the case for mode A, while the bottom image corresponds to mode B.

## References

- <sup>1</sup>BARKLEY, D. & HENDERSON, R. D., (1996), "Three-dimensional Floquet stability analysis of the wake of a circular cylinder", *J. Fluid Mech.* **322**, 215-241.
- <sup>2</sup>WILLIAMSON, C. H. K., (1996), "Vortex dynamics in the cylinder wake", *Annu. Rev. Fluid Mech.* **38**(1), 477-539.
- <sup>3</sup>KARNIADAKIS, G. E., ISRAELI, M., & ORSZAG, S. A., (1991), "High-order splitting methods for the incompressible Navier—Stokes equations", *J. Comput. Phys.* **97**(2), 414-443.
- <sup>4</sup>THOMPSON, M. C., LEWEKE, T. & WILLIAMSON, C. H. K., (2001), "The physical mechanism of transition in bluff body wakes", *J. Fluids Struct.* **15**(3-4), 607-616.

## Brief Biography

Zhi is a PhD student at Monash University investigating the dynamics of transitions in bluff body wakes.