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Sidewall Constraints on Laminar Horizontal Convection in Very Shallow Enclosures

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1. INTRODUCTION

Horizontal convection is a distinctive class of natural convection, where non-uniform heating and cooling occurs along just one horizontal boundary of the enclosure. The research interest in horizontal convection originated from geophysical and geological [1] flows due to the potential to explore ocean overturning circulation. While the manifestation of flow dynamics in horizontal convection under applied thermal forcing is increasingly well understood with a fixed or small range of enclosure aspect ratios [2-4], the characteristics of the key mechanisms under different aspect ratios, and particularly in shallow enclosures relevant to Earth's oceans, is less clear. The present study aims to delineate the Nusselt-Rayleigh number scaling and also explores the bounds of the scaling and flow characteristics on the sidewall of the shallow enclosure horizontal convection.

2. METHODOLOGY

A rectangular enclosure of length L, height, H (aspect ratio, $A = H/L$) is studied. The flow with buoyancy modelled using a Boussinesq approximation is driven by imposing a linear temperature profile on the bottom boundary of the enclosure, and insulating temperature conditions on the remaining boundaries. The 2D incompressible Navier–Stokes equations augmented by a buoyancy term in the momentum equation and a scalar advection-diffusion transport equation for temperature are solved by a high-order in-house solver, which employs a spectral-element method for spatial discretisation and a 3rd-order time integration scheme based on backwards-differencing.

3. RESULTS

Numerical simulations are conducted for a wide range of aspect ratio $10^{-3} \leq A \leq 0.16$ and Rayleigh numbers $10 \leq Ra \leq 10^{17}$ maintaining a fixed Prandtl number $Pr = 6.14$, representative of water. Based on the change of Nusselt number with an increase in Rayleigh number, different regimes can be classified. At low-Ra, the Nu for all A-values are independent of Ra, but scales with the aspect ratio as $Nu \propto A$, and this Ra-range is marked as diffusion-dominated regime. At each aspect ratio, there is a Ra-value beyond which the Nusselt number displays a rapid increase. This regime will be referred to as the transition regime. The onset of the transition regime is demarked by a threshold Rayleigh number Ra_t, with a corresponding Nusselt number, Nu_t. Ra_t is defined when Nusselt number has deviated by 5% from its Ra-independent value. The threshold Rayleigh number was then found to scale as $Ra_t \propto A^{-4}$. Based on the threshold scaling, the $Nu - Ra$ data for all aspect ratios are plotted in figure 1(a) to elucidate the bound of the obtained scaling. For lower values of RaA^4, the corresponding values of Nu/A can be seen to have collapsed onto a universal curve for all aspect ratios. This collapse demonstrates that the flow in this regime is governed by the modified-Ra (RaA^4), which can be defined as $Ra_H = RaA^4 = g\alpha\delta\theta_H H^3/\nu\kappa$. Here, $\delta\theta_H$ refers to the temperature difference along a portion of the bottom boundary of length H. The Ra_H definition reveals that low-aspect ratio horizontal convection is governed by enclosure height rather than its horizontal length. Beyond $\log_{10}(Ra_H) = 1.75$, the Nu/A values for all aspect ratios start branching off with increasing Ra. The horizontal velocity and temperature profiles are extracted for $\log_{10}(Ra_H) = 1.75$ at different horizontal locations at incrementing distances from the hot end wall and are compared against the height from the base. The normalized profiles exhibit a strong collapse to a universal profile, implying self-similarity in velocity fields in this regime. To identify the spread of this regime from the hot end-wall, the normalized velocity data are plotted against different horizontal locations, which are expressed as the distance x' from the hot end-wall in figure 1(b). The result shows that beyond a distance of $4H$ all trends collapse to a single curve, which demonstrates that sidewall effects in diffusion-dominated horizontal convection are confined within $4H$ from the wall. This behaviour emerges only in enclosures having. $A = H/L \leq 1/8 \approx 0.125$, which is shallower than most widely studied enclosure ($A = 0.16$). The extracted temperature profiles also demonstrate the self-similarity feature beyond $4H$ from the hot end-wall. This $4H$ regime has an upper bound of $\log_{10}(Ra_H) = 2.0$.

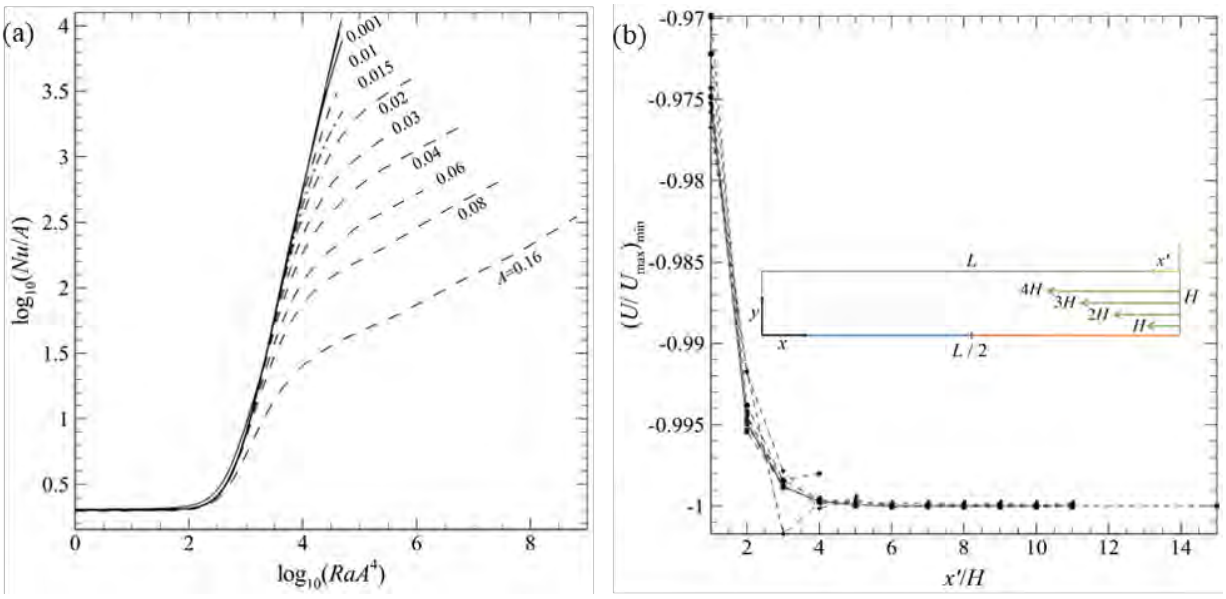

Figure 1. (a) $Nu - Ra$ plot based on threshold scaling and (b) Velocity plotted with distance from hot end-wall. The inset demonstrates a schematic of the enclosure and how different locations were chosen from the hot end-wall.

4. CONCLUSIONS

The obtained threshold scalings inform that shallow-enclosure laminar horizontal convection flow will be insensitive to the enclosure length, and only governed by the height and the horizontal temperature gradient acting over that same scale. The sidewall effects are confined to a region within a distance of approximately four times the enclosure height. These findings reveal that the enclosure in previous investigations was not sufficiently shallow to capture these effects.

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