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The Effect of the Forcing Temperature Profile on Horizontal Convection Flows

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Abstract

Horizontal convection has been used as an idealised model of the ocean overturning circulation, where some non-uniform buoyancy forcing profile is imposed along a horizontal boundary. A high order idealised numerical model is used to investigate the effect of different temperature forcing on the overturning circulation. Time independent, periodic and chaotic regimes are identified in a *Ra*–*n* regime diagram, which shows that a step temperature profile is more unstable than a linear temperature profile. This research highlights the importance of the horizontal buoyancy forcing profile in determining the thermal forcing required to produce instability in horizontal convection. In addition, Nusselt number scales to $Ra^{1/5}$ in the fully convective regime with a trend toward $Ra^{1/4}$ beyond $Ra = 1\times10^{10}$.

Introduction

Heat is transported through fluid media in a number of different ways which produces various physical phenomena as seen in natural systems as well as in engineering applications. One such process is through horizontal convection. Horizontal convection describes the motion of fluids due to temperature differences imposed along a horizontal boundary. Temperature variation across the Earths oceans arising from solar forcing and salinity variation through ice- formation or fresh-water injection from melting ice have a huge impact on the circulation pattern of the ocean. Fundamentally, it is important to understand the effect of changes in temperature forcing on the ocean circulation.

Horizontal convection has been used as an idealised model of the ocean overturning circulation [13, 6, 4] , where some non-uniform buoyancy forcing profile is imposed along a horizontal boundary. The buoyancy forcing profile is representatives of the surface temperature variation of the global ocean due to solar radiation at different latitudes. This motivates the question as to how ocean overturning circulation will be affected by different temperature profiles. Numerous studies of horizontal convection have employed different buoyancy forcing profiles, including linear temperature profiles and combinations of fixed-temperature and fixed heat flux conditions along a horizontal boundary within a rectangular enclosure. Bilgen *et al.*[1] studied convection heat transfer in an enclosure heated sinusoidally but it was heated on a vertical side wall with all other walls insulated. More recently, Khansila *et al.*[5] performed a similar investigation in an enclosure filled with porous medium. To date there has been no comprehensive study into the behaviour of horizontal convection with systematic variation of the imposed buoyancy forcing profile.

Numerically, horizontal convection has been shown to be unsteady and turbulent, capable of maintaining overturning circulation within an enclosure [12, 2, 3]. Three-dimensional simulation [3] suggests that horizontal buoyancy forcing can leads to turbulent circulation at large Rayleigh number and supports Nusselt number scaling to $Ra^{1/5}$ in both laminar and turbulent regimes. However, Sheard *et al.*[11] has shown Nusselt number trending toward a $Ra^{1/4}$ scaling at high Rayleigh number which is in line with a theoretical upper bound of $Ra^{1/3}$ [12].

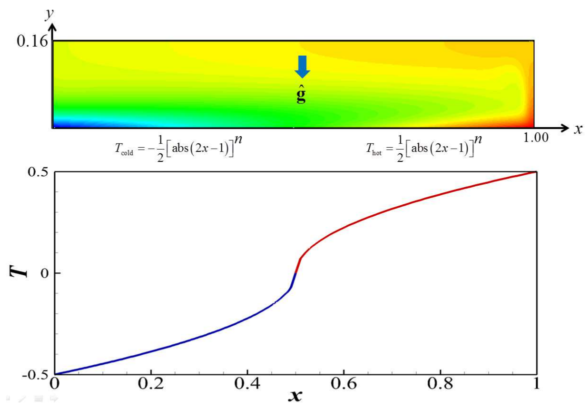


Figure 1. Schematic diagram of the computational domain with a forcing temperature profile along the bottom horizontal boundary. The forcing temperature profiles are as shown.

In this study, a high-order spectral element solver is used to study horizontal convection within a rectangular enclosure with buoyancy forcing variation in the form of different temperature profiles imposed along a horizontal boundary. The controlling parameters in this investigation are the Rayleigh number, *Ra*, and a shape parameter, *n*, controlling the shape of the temperature profile. The shape parameter varies from 0 (describing a step profile) to 1 (describing a linear profile). Rayleigh numbers spanning from a pure conduction regime to a fully convective regime are investigated. Rayleigh number is the key parameter characterizing horizontal convection, it quantifles the strength of thermal forcing on the flow. The onset of unsteady flow is determined by the time dependency of the heat fluxes through the forcing boundary. A Fourier analysis will be used to explain the natural of the unsteady flow at high Rayleigh number. The effects of different temperature profile on the Nusselt number scaling will be investigated.

Numerical Setup

The system comprises a rectangular enclosure of width L and height H. The flow is driven by a non-uniform temperature profile applied along the bottom of the enclosure. The temperature profile is described with a power law equation which can be written as

$$\begin{aligned} T_{\text{cold}} &= -\frac{1}{2}[\text{abs}(2x-1)]^n, \qquad 0 \le x \le 0.5 \\ T_{\text{hot}} &= +\frac{1}{2}[\text{abs}(2x-1)]^n, \qquad 0.5 < x \le 1 \end{aligned} \tag{1}$$

where n varies from $n = 1$ being a linear temperature profile to $n = 0$ for a step temperature profile. The non-dimensionalised temperature varies between -0.5 to 0.5, when the system reaches a thermal equilibrium there is no net heat input across the forcing boundary. An initial temperature of 0 is prescribed for the whole enclosure at the start of the simulation. A schematic diagram of the setup is shown in figure 1.

The side and top walls are insulated (a zero temperature gradient is imposed normal to the wall), and a no-slip condition is imposed on the velocity field on all walls. A Boussinesq approximation for fluid buoyancy is employed, in which density differences in the fluid are neglected except through the gravity term in the momentum equation. Under this approximation the energy equation reduces to a scalar advection-diffusion equation for temperature which is evolved in conjunction with the velocity field. The fluid temperature is related linearly to the density via the thermal expansion coefficient α. The dimensionless Navier–Stokes equations governing a Boussinesq fluid may be written as

$$\frac{\partial \mathbf{u}}{\partial t} = -(\mathbf{u}\cdot\nabla)\mathbf{u} - \nabla p + Pr\nabla^2\mathbf{u} - PrRa\hat{\mathbf{g}}T, \qquad (2)$$

$$\nabla\cdot\mathbf{u} = 0, \qquad (3)$$

$$\frac{\partial T}{\partial t} = -(\mathbf{u}\cdot\nabla)T + \nabla^2 T, \qquad (4)$$

where $\mathbf{u}$, p, t, Ra, Pr, $\hat{\mathbf{g}}$ and T are the velocity vector, kinematic static pressure, time, Rayleigh number, Prandtl number, a unit vector in the direction of gravity, and temperature, respectively. Lengths are scaled by the enclosure width L, velocities by κ_T/L (where κ_T is the thermal diffusivity of the fluid), time by L^2/κ_T, and temperature by δT (the imposed temperature difference imposed across the bottom wall). The horizontal Rayleigh number is defined as

$$Ra = \frac{g\alpha\delta T L^3}{\nu\kappa_T}, \qquad (5)$$

where g is the gravitational a, cceleration and ν is the kinematic viscosity of the fluid. The Prandtl number of the fluid is given as

$$Pr = \nu/\kappa_T, \qquad (6)$$

and throughout this study $Pr = 6.14$, which approximates water at laboratory conditions. The Nusselt number, a measure of the ratio of convective to conductive heat transfer, is defined as

$$Nu = \frac{F_T L}{\rho c_p \kappa_T \delta T}. \qquad (7)$$

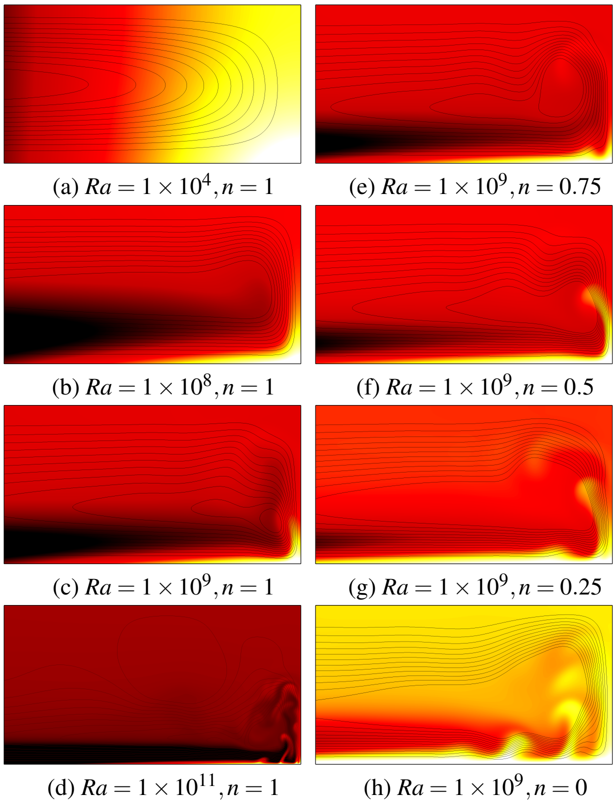
The heat flux is given by

$$F_T = \kappa_T \rho c_p \overline{\frac{\partial T}{\partial y}}, \qquad (8)$$

where c_p is the specific heat capacity of the fluid, and $\overline{\partial T/\partial y}$ is the average absolute vertical temperature gradient along the forcing boundary.

The governing equations (2)-(4) are solved with an in-house solver, Viper, which uses high order spectral element method for spatial discretization and a third-order time integration scheme based on backward-differencing. This code had been validated in a number of studies on wake dynamics of flow over bluff bodies [9, 10].

The computational mesh used in the current study consists of 3692 macro elements with a clustering adjacent to the wall boundaries coupled with a polynomial order of $N = 7$ resulted in a refined mesh which are capable of resolving small scale structures within the boundary layer. A mesh resolution study had been conducted which shows the polynomial order of $N = 7$ to be the most optimum value to be use for the current study.



(a) $Ra = 1\times10^4, n = 1$ (e) $Ra = 1\times10^9, n = 0.75$

(b) $Ra = 1\times10^8, n = 1$ (f) $Ra = 1\times10^9, n = 0.5$

(c) $Ra = 1\times10^9, n = 1$ (g) $Ra = 1\times10^9, n = 0.25$

(d) $Ra = 1\times10^{11}, n = 1$ (h) $Ra = 1\times10^9, n = 0$

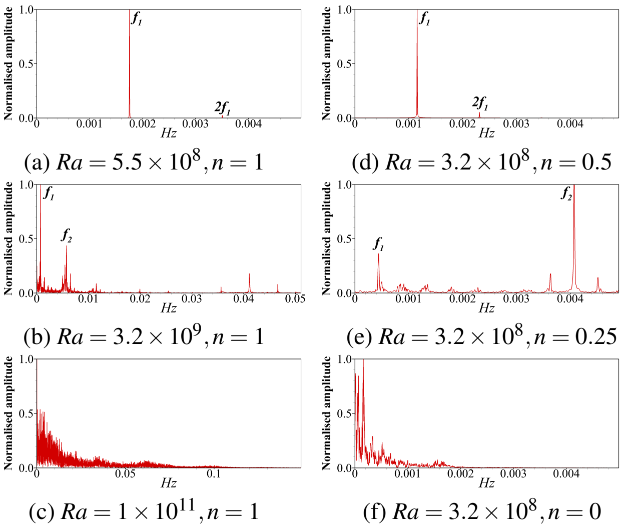
Figure 2. Close-up (30% of the hot end of the enclosure) plot of temperature contour overlayed with streamline at different Rayleigh number, Ra, and forcing temperature profile, n. The left panels show the effect of Rayleigh number with linear thermal forcing at the boundary, the right panels show the effect of different temperature profile at a fixed Rayleigh number.

Result and Discussion

The temperature differential along the forcing boundary compels the fluid to rise at the more buoyant (hotter) end of the enclosure and fall at the less buoyant (colder) end. This creates an overturning circulation within the enclosure. The variation in Rayleigh number serves to increase the intensity of the thermal forcing along the boundary, this is further compounded by the change in the temperature forcing from a linear to a step profile. The structure of the flow can be broken down into three distinctive regimes, namely a conduction dominated regime, a transisitional regime and a convection dominated regime. Within the convection dominated regime, the flow can be further described by a time-independent stable state, a time-periodic state and a time-dependent chaotic state.

Flow Structure

At a low Rayleigh number, the flow structure is mainly conduction dominated with symmetric streamlines as shown in figure 2(a) with a linear forcing temperature profile. As the Rayleigh number is increased, the streamlines begin to cluster toward the forcing boundary and the end wall to create a plume-like jet at the end wall at the hot end of the enclosure, (figure 2(b)). This jet transports heat convectively into the enclosure to create a more convectively dominated structure. With



(a) $Ra = 5.5 \times 10^8, n = 1$
(d) $Ra = 3.2 \times 10^8, n = 0.5$
(b) $Ra = 3.2 \times 10^9, n = 1$
(e) $Ra = 3.2 \times 10^8, n = 0.25$
(c) $Ra = 1 \times 10^{11}, n = 1$
(f) $Ra = 3.2 \times 10^8, n = 0$

Figure 3. Fast Fourier frequency analysis of the Nusselt number along the forcing boundary for various Rayleigh number and forcing temperature profiles to illustrate the nature of a time-periodic and chaotic flow. The amplitude is normalised by the local maximum amplitude of the flow.

a further increase in the Rayleigh number, the convective forcing is sufficient to break out of the thermal boundary layer to create plume like structures near the end wall which give rise to time-periodic and a time-dependent chaotic states as shown in figure 2(c) and (d). Multiple plumes are observed in the time-dependent chaotic state which create a strong overturning circulation over the whole computational domain.

The right panels of figure 2 show the effect of different thermal forcing profiles at $Ra = 1 \times 10^9$. As the forcing temperature profile changes from a linear to a step profile, the single vertical plume attached to the end-wall is supplanted by plume eruptions in advance of the end-wall. The circulation also changes from a small, localised behaviour (figure 2(e)) to a more global and larger structure as shown in figure 2(h). These multiple plumes are found to significantly affect the periodicity of the flow, as discussed in the following section.

Flow Periodicity

Fourier analysis of Nusselt number time history data is used to characterize the periodicity of the flow. In order to obtain statistical meaningful representation of the flow field, sufficient integration time is used to ensure saturation of the solutions, both at the on set of periodic and time-dependent chaotic regimes. At the onset of the convection-dominated regime, $Ra > 1 \times 10^8$, the Nusselt number and the flow are time-independent stable state. The time periodic nature of the flow develops at $Ra \gtrsim 5 \times 10^8$ for the linear temperature profile. The Fourier frequency analysis for $Ra = 5.5 \times 10^8$ with linear temperature profile, shows that the flow has a dominant frequency of $f_1 = 1.752 \times 10^{-3} Hz$ with a weaker harmonic peak at $2f_1$ as shown in figure 3(a). At a highger Rayleigh number of $Ra = 3.2 \times 10^9$, the convection dominated regime enters into a quasi-periodic state with two incommensurate frequencies of $f_1 = 7.159 \times 10^{-4} Hz$ and $f_2 = 5.803 \times 10^{-3} Hz$, and many of their linear combination frequencies with some noise appearing in the spectrum in figure 3(b). At $Ra = 1 \times 10^{11}$ the flow transitions to a non-periodic state with more pronounced broadband noise without any noticeable dominant frequency. This is shown in figure 3(c).

When the thermal forcing profile is steepened towards a step

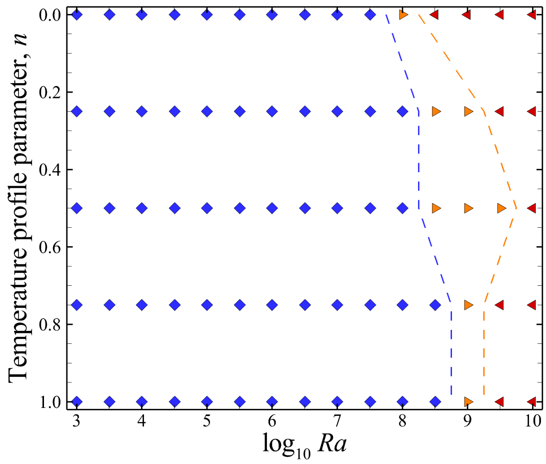


Figure 4. A regime diagram in the $Ra - n$ space to illustrate the time-independent, time periodic and chaotic regime of the flow structures.

profile, the onset of unsteady periodic flow occurs at a lower Rayleigh number $Ra = 3.2 \times 10^8$ for $n = 0.5$, and the flow becomes quasi-periodic then chaotic at profiles with $n = 0.25$ and $n = 0$ (step), respectively. These are shown in figure 3(d)-(f). With an increasing thermal gradient applied to the forcing boundary, the flow becomes increasingly unstable and enters into a time-dependent chaotic state.

Figure 4 shows a regime diagram in the $Ra - n$ parameter space, where it is shown that as the temperature gradient departs from the linear profile towards a step profile, the onset of periodic/chaotic flows occur at a lower Rayleigh number. However, at intermediate profile shapes ($n \approx 0.5$), the transition from periodic to irregular flow is delayed, leading to a wider range of Rayleigh numbers over which time-periodic flow is obtained.

Scaling at High Rayleigh Number

Rossby [7] had shown that circulation can indeed be maintained when heating is applied along a single horizontal boundary with the Nusselt number scaling with $Ra^{1/5}$ as reported in a number of previous studies [6, 8, 3]. However, Siggers *et al.*[12] argued that there is an asymptotic upper bound on Nusselt number to scaling with $Ra^{1/3}$. Figure 5 shows a plot of base-10 logarithms of Nusselt number against Rayleigh number for different forcing temperature profiles. In the case of time-periodic and time-dependent simulation, a statistical mean Nusselt number is obtained from the time series data. Our simulations show that there is a strong agreement with the Rossby scaling in the convection dominated regime up to the onset of the time-periodic state. Beyond this the scaling factor increased toward a factor of 0.25 for the temperature profile parameter of $n = 1$ (linear), 0.75 and 0.5. The $n = 0$ (step) and $n = 0.25$ profile have a smaller increase in the scaling factor to $\gamma = 0.215$. This finding supports the results of Sheard *et al.*[11], which observed a trend approaching the upper-bound value of Siggers [12].

Conclusion

Horizontal convection under a systematic variation in the forcing temperature along a horizontal boundary is investigated using a high resolution in-house spectral element code. The flow dynamics, periodicity and heat transfer scaling are presented. It is established that for all forcing profiles considered, at sufficiently high Rayleigh number, heat transfer is dominated by convection where plume-like structures break out of the thermal

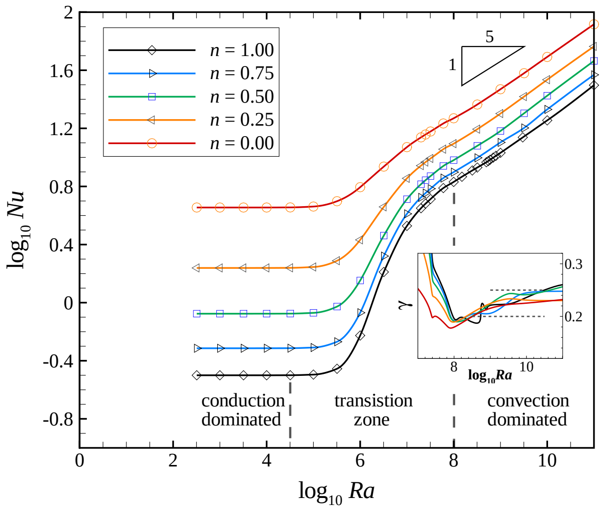


Figure 5. A plot of Nusselt number, Nu, against Rayleigh number, Ra, in a base 10 logarithmic scale for different forcing temperature profile. The inset diagram shows local gradient, $\gamma = \mathrm{d}[log(Nu)]/\mathrm{d}[log(Ra)]$ plotted against the Rayleigh number. The Rayleigh numbers are divided into three zones, namely conduction dominated, transition zone and convection dominated zone.

boundary layer, establishing a time periodic instability within the system. The periodic regime quickly gives way to a chaotic behaviour with further increases in Rayleigh number.

Within the time-periodic convection dominated regime, the Nusselt number scales with $Ra^{1/5}$ as observed by previous investiators, [6, 8, 3]. However, an upward trend in the scaling factor was obtained in the chaotic convection dominated regime. This research highlights the importance of the horizontal buoyancy forcing profile in determining the thermal forcing required to produce instability in horizontal convection.

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